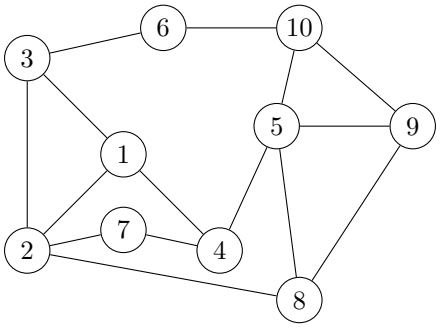
Homework 4: Graphs

You may work in groups, but you must write solutions yourself. List your collaborators at the top of your submission.

Instructions for submission: Submit your homework as a single PDF file to the course Gradescope page. Either scan your *legible* handwritten solutions or type them up in LaTeX.

Problem 1. Consider the following graph G with ten vertices:



(a) What is the distance between vertices 1 and 8?

(b) **BFS I**

(i) List the order in which the vertices are visited when we run a breadth-first search starting from vertex 1. Assume that we visit neighbors in *increasing* order of their labels.

(ii) How many layers does the BFS tree have? **Hint**: L_0 counts as a layer.

(iii) Which vertices are in L_2?

(c) **BFS II**

(i) List the order in which the vertices are visited when we run a breadth-first search starting from vertex 1. Assume that we visit neighbors in *decreasing* order of their labels.

(ii) How many layers does the BFS tree have?

(iii) Which vertices are in L_2?

(d) **DFS**: List the order in which the vertices are visited (i.e. explored) when we run a depth-first search starting from vertex 1. Assume that we visit neighbors in *increasing* order of their labels.

Problem 2. In lecture, we showed that if a graph G has an odd cycle, then it cannot be bipartite. We proved this using a coloring argument, where each of the two colors represents a partition. This showed that if a graph is bipartite, then it has no odd cycles. Prove the converse: if a graph G has no odd cycles, then G is bipartite. **Hint**: use the BFS tree to construct the two partitions into even layers and odd layers.